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AUTHOR	Arbaugh, Fran; Brown, Catherine A.
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ABSTRACT

The research reported in this article is apart of a larger research project designed to investigate ways in which participation in a particular study group influenced inservice secondary mathematics teachers' knowledge, thinking, and practice regarding the teaching of mathematics. This paper examines how a particular framework for considering attributes of mathematical tasks influenced the members of the study group. This paper specifically addresses the following questions: (1) In what activities did the members of the study group engage regarding the Levels of Cognitive Demand criteria of the Mathematical Tasks Framework? and (2) In what ways did learning about the Levels of Cognitive Demand criteria of the Mathematical Tasks Framework influence these teachers' knowledge about and use of high-level tasks in their geometry classes? Results reported in this paper indicate that work with the Levels of Cognitive Demand influenced how the participant teachers thought about instructional planning and implementation issues in the following manners: (1) the teachers began to think more deeply about the relationship between tasks and levels of student thinking (or reasoning); and (2) some of the teachers began to select and incorporate more high-level tasks into their teaching practice. (Contains 25 references.) (MM)

RUNNING HEAD: INFLUENCES OF THE MATHEMATICAL TASKS FRAMEWORK

Influences of the Mathematical Tasks Framework on High School Mathematics Teachers' Knowledge, Thinking, and Teaching

Fran Arbaugh

University of Missouri-Columbia

Catherine A. Brown

Indiana University-Bloomington

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Introduction

The relationship between the types of tasks students engage in when learning mathematics and the mathematics they learn has been a subject of research that has spanned many years (see, for example, Hiebert & Wearne, 1993; Marx & Walsh, 1988; Stein & Lane, 1996). NCTM (1991) recognized the importance of this relationship in making its recommendations regarding mathematics teaching; one of the six standards of mathematics teaching contained recommendations for teachers about choosing and implementing "worthwhile mathematical tasks" (p. 25). NCTM (2000) continued to emphasize the importance of mathematical tasks in *Principles and Standards for School Mathematics*:

> In effective teaching, worthwhile mathematical tasks are used to introduce important mathematical ideas and to engage and challenge students intellectually. Well-chosen tasks can pique students' curiosity and draw them into mathematics.... Regardless of the context, worthwhile tasks should be intriguing; with a level of challenge that invites speculation and hard work. (p. 16-17)

It is generally argued that mathematics students learn the types of mathematics inherent in the tasks that they complete:

> Tasks that ask students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking; tasks that require students to think conceptually and that stimulate students to make connections lead to a different set of opportunities for student thinking. (Stein & Smith, 1998)

Given the influence tasks have on the learning of mathematics, it is important that teachers understand this relationship. As Ball writes in the Forward of *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development* (Stein, Smith, Henningsen, & Silver, 2000), "Acquiring the ability to think with precision about mathematical tasks and their use in class can equip teachers with more developed skills in the ways they select, modify, and enact mathematical tasks with their students" (p. xii). These skills are important ones for teachers to develop, so much so that Ball considers them to be "a core domain of teachers' work" (p. xii).

The research reported in this article is part of a larger research project designed to investigate the ways in which participation in a particular study group influenced inservice secondary mathematics teachers' knowledge, thinking, and practice regarding the teaching of mathematics (Arbaugh, 2000). Other papers resulting from this research project focus on the mechanism for the professional development – the use of a study group (for example, Arbaugh, in review). In this paper, we examine how learning about a particular framework for considering attributes of mathematical tasks influenced the members of the study group. This paper specifically addresses the following questions:

- In what activities did the members of the study group engage regarding the Levels of Cognitive Demand criteria of the Mathematical Tasks Framework?
- In what ways did learning about the Levels of Cognitive Demand criteria of the Mathematical Tasks Framework influence these teachers' knowledge about and use of high-level tasks in their geometry classes?

The Mathematical Tasks Framework

In the 1990's, the QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) project[1] began to conduct research designed to study the relationship between mathematical tasks and student learning. Using the works of Doyle (1983,

1988) and Hiebert and Wearne (1993) as theoretical background, the QUASAR researchers conducted many studies designed to understand task use in middle-school mathematics classrooms. The researchers examined the relationships between tasks as they are written, set up and then implemented in the classroom, and student learning (Stein & Lane, 1996). They developed a framework for distinguishing the levels of cognitive demand required by different types of mathematical tasks (Smith & Stein, 1998; Stein & Lane, 1996). They developed a framework for considering factors associated with the maintenance and decline of high-level tasks during implementation (Henningsen & Stein, 1997; Stein & Smith, 1998). They combined all of this research to propose recommendations for the professional development of mathematics teachers (Stein et al., 2000). The QUASAR researchers developed a framework much in the spirit of recommendations made by Marx and Walsh (1988):

> Our major theme has been that the improvement for instruction rests to a large degree on a better understanding of the complex nature of classroom work. Such an understanding must begin with a descriptive theory of academic work, a theory that embraces the conditions, products, and cognitive plans that are the major constituents of this work. Ultimately, of course, a coherent theory of classroom tasks must lead to guidelines for instruction.... Teachers must also structure the setting and social context in which a task is completed to ensure that these factors do not sabotage the teacher's intent for the task. (p. 217)

The Mathematical Task Framework (MTF) describes the phases through which tasks pass as they are implemented in a mathematics classroom. This framework is depicted in Figure 1 and explained below.

---Insert Figure 1 about here---

In describing the framework, Stein, Grover, and Henningsen (1996) write: "Instructional tasks are seen as passing through three phases: first, as curricular materials; second, as set up by the

[1] QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) is a Ford Foundation-funded project "aimed at assisting schools in economically disadvantaged communities to develop instructional

teacher in the classroom; and third, as implemented by students during the lesson" (p. 460). In the first phase, consideration is given to the level of cognitive demand required by mathematical tasks as written in curricular materials. For example, tasks that ask students to memorize a fact or to perform an algorithm rotely encourage a certain type of mathematical thinking. Tasks that ask students to look for patterns, generalize, make connections, or think conceptually encourage a different kind of thinking (Stein & Smith, 1998).

Smith, Grover, and Henningsen (1996) describe task set up and implementation:

> *Task set up* is defined as the task that is announced by the teacher. It can be quite elaborate, including verbal directions, distribution of various materials and tools, and lengthy discussions of what is expected. Task set up can also be as short as simply telling students to begin work on a set of problems displayed on the blackboard. *Task implementation*, on the other hand, is defined by the manner in which students actually work on the task. Do they carry out the task as it was set up? Or is the task somehow altered in the process of working through it? (p. 460, emphasis in original).

Stein and Lane (1996) argue that when QUASAR project teachers chose high level tasks for use in their classrooms, set them up and implemented the tasks at a high level, there was an increase in student understanding and reasoning. Henningsen and Stein (1997), interested in understanding how high level tasks "lose their punch," argue that certain classroom factors are associated with the maintenance, or decline, of high-level cognitive demands as tasks pass through the Mathematical Tasks Framework.

The larger research project from which this particular study originates (Arbaugh, 2000) utilizes the Mathematical Tasks Framework in its entirety, considering its attributes as a framework for professional development as well as its use as a framework for research. In this paper we focus solely on the way in which a group of teachers learned the Levels of Cognitive

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programs that emphasize thinking, reasoning, and problem solving in mathematics" (Stein & Lane, 1996).

Demand criteria, and the influences of that learning on their use of high-level tasks in their geometry classes. Thus, a closer examination of the Levels of Cognitive Demand follows.

Levels of Cognitive Demand.

The first phase of the Mathematical Tasks Framework is entitled "Tasks as they appear in curricular/instructional materials" (see Figure 1). In order to categorize tasks, QUASAR project created the Levels of Cognitive Demand (see Table 1). The researchers analyzed hundreds of tasks that were being used in project classrooms and from that analysis created a set of criteria that "when applied to a mathematical task (in print form)...can serve as a judgment template (a kind of scoring rubric) that permits a 'rating' of the task based on the kind of thinking it demands of students" (Stein et al., 2000, p. 15).

----Insert Table 1 about here---

The Study

The "Toyota Time Study Group"

Establishment of the study group. In spring of 1999, the geometry teachers at Ericson Valley High School (EVHS) were awarded a Toyota Time Grant (sponsored by the National Council of Teachers of Mathematics) to support the development and implementation of a new geometry curriculum focused on the reasoning level used by students. They identified two areas on which to focus their reform efforts: pedagogy and the curriculum. Recognizing that change is often a difficult process to undertake and implement, and that support is a critical component of change (Fennema & Nelson, 1997), the geometry teachers at EVHS welcomed the opportunity to participate in a professional development experience that would help them focus on the tasks that they use in their geometry classes.

In the fall of 1999, all of the geometry teachers at EVHS were invited to join the "Toyota Time Study Group." Eight of the nine geometry teachers at EVHS (and one teacher from another high school in the district who had heard about the group) expressed interest in being involved, and meetings began in October, 1999.

Study group meetings. The study group met ten times from October 1999 through March 2000 (approximately once every two weeks). The majority of study group meetings took place at the high school; the group met once at a local university and once at a study group member's home. District funds (obtained from matching funds for the Toyota Time Grant) were used to support the teachers with release time for meetings held during the school day and with stipend pay for time that exceeded contract hours. The one non-EVHS teacher was supported by his own principal.

Study Participants

Seven of the nine original members of the study group participated in the research reported in this paper.[2] All seven participants were high school mathematics teachers who taught at least one section of geometry during the year in which this research occurred. (Alphabetically, their names are: Annie, Brian, Carl, Craig, Ed, Megan, and Pamela.)[3] At the time of the study, these teachers taught at Ericson Valley High School (school population of approximately 1750 students with 16 mathematics teachers during the year of the study). Five of the seven teachers (Annie, Brian, Craig, Ed, and Pamela) taught the same level geometry course, and used the same

[2] Due to circumstances not related to the study group itself, two teachers did not complete data collection activities, leaving seven teachers who participated in this aspect of the research.

textbook. Carl taught a "low-level" geometry course, and Megan taught a Geometry course designed for students in an alternative program within the high school. These two teachers used the same textbook in their classes (Serra, 1998), but a different book than other study group members, who used a more "traditional" textbook.

These seven teachers had varied teaching experience. Two of the teachers were in their first year of teaching (Annie and Pamela), having recently completed bachelors degrees in mathematics education. The other five teachers had the following number of years experience teaching mathematics: 3 (Craig), 8 (Ed), 15 (Megan), 24 (Brian), and 32 (Carl). Although Craig and Ed had previous teaching experience, they, like Annie and Pamela, were new to EVHS. Craig was new to the area, having taught high school math in a neighboring state. Ed, on the other hand, had recently transferred to EVHS from a middle school in the same district. Brian had also taught at the same middle school, having transferred to EVHS three years prior. Megan and Carl could be considered the "old-timers" of the group, each having taught at EVHS for the majority of their careers. Additionally, during the year of this study, Brian was in his first year as the mathematics department chair.

The first author of this paper was also a member of this study group, taking the role of group participant/facilitator and researcher, thus having an active role in study group meetings. She played the role of "expert other" -- asking probing questions and challenging the teachers to verbally reflect on their knowledge and teaching. She found articles for the group to read based on the requests from the teachers. After receiving feedback from the teachers on what they wanted to accomplish at the next meeting, she and Brian, working together, "set the agenda."

[3] All teachers' names, as well as the name of the high school in which they taught, are pseudonyms.

The teachers completed written reflections at her request. As primary researcher, she collected all data necessary to complete the study.

Data Collection and Analysis

For the larger research project, multiple sources of data were collected in order to support data analysis and subsequent research conclusions, as well as to strengthen analysis through triangulation (Ernest, 1997). For this particular study, pertinent data sources include: 1) audio-taped accounts of study group meetings; 2) two audio-taped teacher interviews (initial interview and final interview); 3) artifacts from teacher interviews; 4) all tasks that teachers used in their classrooms during the first and final weeks of the study; and 5) tasks that teachers used in their classrooms to supplement the adopted text over the course of the study. Analysis of these data is explained below.

Analysis of study group meeting audio tapes. Whole group discussions of each study group meeting were audio-taped and transcribed for analysis. Transcripts were coded in two different "passes," following the advice given by Coffey and Atkinson (1996) who write:

> the term *coding* encompasses a variety of approaches to and ways of organizing qualitative data. As parts of an analytical process, however, attaching codes to data and generating concepts have important functions in enabling us rigorously to review what our data are saying. (p. 27; authors' emphasis)

Further, they write:

> In practice, coding usually is a mixture of data reduction and data complication. Coding generally is used to break up and segment the data into simpler, general categories *and* is used to expand and tease out the data in order to formulate new questions and levels of interpretation. (p. 30; authors' emphasis)

The codes for the first pass on all of the transcripts were determined by the research questions (which were framed by the Mathematics Task Framework) and were used primarily for data reduction purposes. The codes for the second pass emerged from the data as a result of looking for patterns and trends to support interpretation.

Analysis of audio-taped interviews. The teachers participated in two interviews – an initial interview that occurred before study group meetings began and a final interview that occurred after the meetings had concluded. Coding of the transcribed audio-tapes from each interview was undertaken in a similar manner as described above. Once coded, the data were reduced by creating packets of discussion segments that had been similarly coded. The packets were then used to interpret the data and search for patterns.

Analysis of teacher-generated interview artifacts. One of the methods we used to collect data about the teachers' knowledge of the Levels of Cognitive Demand was to have them complete the same task-sorting activity as a part of both the initial and final interviews. The task-sorting activity used in both interviews was designed by the QUASAR project (Stein et al., 2000). Prior to the initial and final interview, the teachers completed two different task-sorts, sorting 20 middle-school level tasks into categories of their own making (see Table 2 for instructions given to teachers and Appendix A for the 20 tasks). During the subsequent interview, teachers were asked to explain their categories and what tasks they placed in each category.

---Insert Table 2 about here---

Analysis was conducted in the following manner. Data were physically organized to facilitate a vertical analysis (by interview) and a horizontal analysis (across the two interviews). Data from the initial interview task-sorts were then examined and five emergent "groups" were created, so as to "chunk" the data (see Table 3). Data from the final interview task-sorts were then categorized using the emergent groups from the initial interview task-sort analysis. As indicated in Table 3, a new group emerged in analysis of the final interview task-sorts and two groups were no longer applicable.

---Insert Table 3 about here---

Further analysis was conducted on the final interview task-sorts. Five of the seven teachers created categories based on the Levels of Cognitive Demand. Their placement of tasks within the Levels of Cognitive Demand categories were assessed based on QUASAR-provided "answers" regarding categorization of 20 tasks (Stein et al., 2000).

Analysis of tasks used in teachers' classrooms. Twice during the study, once at the beginning and once at the end, data were collected regarding all of the tasks teachers used in one week in their classrooms. The tasks from the two different weeks were categorized using the Levels of Cognitive Demand criteria. Once categorized, analysis included examining each teacher's tasks, comparing task use at the beginning of the study with tasks used at the end of the study.

Data were also collected in the form of tasks the teachers used to supplement their adopted texts over the course of the study. They were given a folder to keep on their desks in which to collect supplemental tasks, and then they brought these folders to subsequent study group meetings. These tasks were analyzed with regard to Level of Cognitive Demand.

Results and Discussion

Study Group Activities Involving the Levels of Cognitive Demand Criteria

For the first two meetings, the teachers engaged in activities to help them understand the Levels of Cognitive Demand criteria. During the first study group meeting, the teachers read Smith and Stein (1998), an article that provides a short, but thorough introduction to the Mathematical Tasks Framework. Following a whole-group discussion of the article, the teachers engaged in a task-sorting activity designed to help them learn the levels of cognitive demand criteria.[4]

Stein et al. (2000) recommend that

> one way we have found to help teachers learn to differentiate levels of cognitive demand is through the use of a task-sorting activity. The long-term goal of this activity is to raise teachers' awareness of how mathematical tasks differ with respect to their levels of cognitive demand, thereby allowing them to better match tasks to goals for student learning. (p. 18)

The teachers worked in pairs to sort twenty cards, each card containing a different high school level mathematical task. The teachers were given explicit criteria (see Table 1) for categorizing the tasks according to the required level of cognitive demand. They spent about 25 minutes completing the sort, after which they engaged in a whole-group discussion about their categorizations that lasted into the next study group meeting. According to Stein et al. (2000)

> The benefits of a task-sorting activity…accrue not simply from completing the sort, but rather from a combination of small- and large-group discussions that provide the opportunity for conversation that moves back and forth between specific tasks and the characteristics of each category…and negotiating definitions for the categories. We have

[4] This task-sorting activity, containing tasks appropriate for use in high school mathematics, was designed by the authors to emulate the task-sorting activity described in Stein et al., 2000. Specifics about activity design and results of design justification can be found in Arbaugh (2000) and Arbaugh & Brown (in review).

> found that participants do not always agree with each other – or with us—on how tasks should be categorized, but that both agreement and disagreement can be productive. (p. 20)

The task-sorting activity was the main activity in which the teachers engaged to learn about the levels of cognitive demand. Other activities in subsequent study group meetings supported the teachers in continuing to consider the levels of cognitive demand required by the tasks they were using in their classrooms.

For example, the teachers spent a greater part of one study group meeting looking through resources to find high-level tasks that would "fit into" their curriculum. At another meeting, teachers were challenged to consider "tweaking" a low-level task so as to raise the cognitive demand it required of students. Teachers reflected in writing about the level of tasks they were using in their classroom. The Levels of Cognitive Demand criteria permeated many study group discussions.

Influences of Learning about the Levels of Cognitive Demand

In the following sections, we discuss two analyses: of the 1) interview task-sorts; and 2) the tasks used in the teachers classrooms.

The Interview Task Sorts

As a reminder to the reader, before both the initial and final interviews each teacher sorted 20 middle-school based mathematical tasks into categories of their own making, and then resorted the same tasks in a different way (for a total of 4 task-sorts over the course of the two interviews). In the following discussion, we call the sorts that the teachers completed prior to the initial interview *Initial Sort 1* and *Initial Sort 2*; we call the sorts that the teachers completed prior to their final interview *Final Sort 1* and *Final Sort 2*. (See Table 4)

---Insert Table 4 about here---

We first discuss results of *Initial Sorts 1* and 2, followed be a discussion of *Final Sorts 1* and 2. These sections are followed by a discussion in which the initial and final interview task-sorts are compared.

Initial interview task-sorts (*Initial Sorts 1* and 2). The teachers created a total of fifty-four categories in fourteen different sorts during *Initial Sorts 1* and 2 (See Figure 2).

---Insert Figure 2 about here---

The 54 teacher-created categories were organized into five larger emergent groups as shown in Table 5. Each group is described below.

---Insert Table 5 about here---

"Categories based on mathematical content or topics" contains 16 teacher-created categories such as "Percents," "Common Fractions," "Algebra," and "Comparing Numbers." The name of this emergent group is relatively self-explanatory. As Megan said in explaining how she named her category, "I thought, this is what I do with my algebra class; this is something I can do on problem solving." Carl separated out all of the tasks dealing with decimals, fractions, and percents before creating other categories in his *Initial Sort 1*.

"Categories based on what the students have to do to complete the task" contains 3 teacher-created categories that dealt with student processes while doing tasks. For example, when describing his category "setting up problem/providing explanation," Ed talked about how these tasks required that the students "think a little bit more than just picking out numbers and doing a computation problem." Brian explained "computation without a calculator" by describing Task D (see Appendix A): "Evaluate the expression—so they plug in three into all those different things and calculate the answer."

"Categories based on what students have to do to answer the task" includes items with which the teachers were concerned about the final step of the problem—the answer. For example, Brian's entire *Initial Sort 2* was grounded in the kind of response that is required of the students. He made a distinction among those tasks that require students to explain their thinking or justify their response, those tasks that require students to respond but may not focus on student reasoning, and those tasks that require short answer responses.

"Categories based on surface characteristics of tasks as presented" describes teacher-created categories in which the teachers placed items based solely on surface characteristics of the tasks (written text or the visual "look" of the task overall). For example, Pamela's dichotomy of "real-world problems" and "non-real-world problems" was based solely on the text of the tasks. Craig's trichotomy of "story problems without visuals," "problems involving manipulatives or visuals," and "concept practice problems" was based on surface characteristics of the tasks, how they *looked* on the card.

It is interesting to note that a few teacher-created categories with the same, or very similar, names ended up in different emergent groups. For example, two teachers had a category entitled "patterns" which were placed in two different larger groups: "categories based on mathematical content or topics" and "categories based on surface characteristics of the tasks as written." The placement of this particularly named teacher-created category into two different emergent groups was dictated by the teachers' explanations of their categories. When Megan talked about "patterns" it was in the context of patterns as a mathematical content. When Craig talked about assigning Task B (see Appendix A) he said, "well, we have our picture pattern for B," referring to the picture (surface characteristic) of the problem as criteria for assigning it to his "patterns" category.

"Other" contains 3 teacher-created categories. Of the three categories assigned to this emergent group, two are actually named "other" and the third is "mathematical writing."

Final interview task-sorts (*Final Sorts 1 and 2*). The teachers created a total of thirty-six categories in eleven sorts (See Figure 3). Note that, as indicated in the methods section of this paper, Ed, Megan, and Pamela completed only one task-sort for their final interview.

---Insert Figure 3 about here---

The 34 teacher-created categories were assigned to the same groups that emerged from analysis on the *Initial Sorts 1* and 2 with the following exceptions: 1) No teacher-created category fit the criteria for "categories based on mathematical content or topics," thus that group does not exist for *Final Sorts 1* and 2; 2) The "other" emergent group was no longer necessary; and 3) A new group emerged -- "categories based on the Levels of Cognitive Demand." This new emergent group joins 3 of the groups from the analysis of *Initial Sorts 1* and 2: "categories based on what students have to do to complete the task," "categories based on what students have to do to answer the task," and "categories based on surface characteristics of the task as written" to complete the set of 4 groups for *Final Sorts 1* and 2 (see Table 6).

---Insert Table 6 about here---

One further analysis was conducted on the data from *Final Sorts 1* and 2. As stated earlier in this paper, five of the seven teachers created categories in their final interview task-sorts based on the Levels of Cognitive Demand. Just the fact that they used the Levels of Cognitive Demand does not indicate how well they used them. The following is an assessment of these teachers' use of the Levels, based on the QUASAR project's "answers" to the card-sorting activity (Stein et al., 2000).

Brian, Craig, and Megan all used four Levels in one of their final task-sorts; their categories were taken directly from the Levels of Cognitive Demand. Brian said, "I figured I would try to do this sort [based on] the one we did early on in the study group." Craig talked about using categories based on "the sheet we had," referring to the piece of paper he received at the first study group meeting containing the Levels of Cognitive Demand (see Table 1). Megan said, "it just seemed logical, this time, to...fit them into four different levels that we talked about of thinking."

Ed and Carl used three Levels. While based on the Levels of Cognitive Demand, both Carl and Ed adapted QUASAR's work a little bit. Ed explained how he thought about the three levels:

> I just did low meaning basically following an algorithm or a pattern or something that you've been shown how to do, straight computation. Higher...demand—had to put some thought into reading the question, understanding it and the question asked more than just, what is the answer....And these were somewhere in the middle where I thought they were a little higher level than this, but they didn't ask for quite the in-depth.

Carl explained his three levels in a similar manner. To describe "lower level" he said, "they were ones that were algorithms, they just follow the rules." To describe "middle level" he said, "they not only had to work [it] out, they had to explain." He picked Task B (see Appendix A) to describe how he was thinking about "higher level" and said:

> They had to find this, they had to go to the next step and draw the figure and then they had to write an inscription (sic) of telling how you would follow that. So they not only had to know what was going on, they had to be able to describe it so someone else would understand it.

Whether these teachers used three levels or four levels, analysis indicates that most of them had little trouble distinguishing between "high level" and "low level" tasks as defined by the Levels of Cognitive Demand. For example, Craig used two levels (high and low) in *Final Sort 1* and then split those piles into four levels (lowest, low, high, highest) for *Final Sort 2*. (Note: see Appendix A for all tasks described below.) He misplaced only two tasks in *Final Sort 1*, N and R, each belonging in the other category. Ed, in his use of three levels (low, middle, and high) was able to classify all but two of the tasks QUASAR considers to be low level (memorization and procedures without connections) into his "low" category; he missed N and I, and those he placed in his "middle" category. He picked out five of the six tasks that QUASAR considers to be "doing mathematics" and assigned them to his "high category" with only one task (H) placed there that was not "doing mathematics" (and Task H is considered "high level" by QUASAR). Megan, in using four levels, misplaced only one task (N) in making the distinction between high level (doing mathematics and procedures with connections) and low level (procedures without connections and memorization). Brian had the most difficulty with task placement, although he did assign all of the QUASAR-rated low level tasks to the correct categories. He had more trouble with placing high-level tasks into lower level categories. For example, five of the tasks that he said were "procedures without connections" were rated by

QUASAR as being "procedures with connections," and three of the tasks that he assessed as "procedures with connections" were rated by QUASAR as belonging in the "doing mathematics" category.

A closer examination of these data indicated that these teachers had more difficulty making finer distinctions between the sub-categories of low-level tasks (memorization and procedures without connections) as well as the sub-categories of high-level tasks (procedures with connections and doing mathematics). Although Megan only misplaced one task when making the distinction between high- and low- level tasks, she was considerably less able to make the finer distinctions. Both of the tasks that she placed in "memorization" (D and E) belong in the "procedures without connections" category. Likewise, two of the tasks she placed in "procedures without connections" (F and O) belong in "memorization." She had similar problems with "procedures with connections" (M and T should be in "doing mathematics"), and "doing mathematics" (A, C, H, and L should be in "procedures with connections").

<u>Comparing the initial and final interview task-sorts</u>. Many interesting differences exist between the teachers' initial and final interview task sorts. One of those differences is the types of teacher-created categories that were assigned to the emergent group entitled "categories based on what students have to do to complete the task." In *Initial Sorts 1* and 2, every one of these teacher-created categories dealt with student *actions*. For example, "computation" implies the *action* of computing. "Setting up the problem/providing explanation" are *actions*. Compare that to the teacher-created categories that fell into this group for *Final Sorts 1* and 2: "The problem requires translation/representation between two or more modes" and "understanding in a method that you could explain so other[s] understand." While these are also action (verb) based, the

verbs pertain more to an *active thinker* (see Van de Walle, 1994, p. 9-10 for a discussion of "The Verbs of Doing Mathematics").

A second difference that came to light during analysis was the overall trend of how the teachers were considering placement of the tasks. In *Initial Sorts 1* and *2*, the teachers, overall, based many of their categories on surface characteristics of the tasks as written. In addition to the twenty-seven teacher created categories assigned to the emergent group based on "surface characteristics," one could argue that the categories placed in the "mathematical content" group are also based on surface characteristics. Those two groups together include approximately 80% (43/54) of the teacher-created categories. Teachers gave little consideration to students or student *thinking* in *Initial Sorts 1* and *2*. In comparison, twenty out of the thirty-six teacher-created categories in *Final Sorts 1* and *2* were directly related to the Levels of Cognitive Demand criteria. Further, all of the teacher-created categories in the group entitled "based on what students have to do to answer the task" dealt explicitly with student *thinking*. The number of teacher-created categories included in the group based on "surface characteristics" fell to seven out of thirty-six, and no teacher based either *Final Sort 1* or *2* on the mathematical content of the task.

Based on the two differences described above, we can argue that the teachers in this group showed more concern with the relationship that tasks have with student thinking (or reasoning) at the end of the study than at the beginning of the study. Although some evidence exists from the initial interview task sorts that a couple of teachers were reflecting on student thinking (take for example Brian's categories dealing with student responses and evidence of thinking), a majority of *Final Sort 1* and *2* teacher-created categories dealt directly with the level of student thought required by the tasks (29/36 with the combination of items from three different groups). Carl described one aspect of his personal growth in this area:

> The first time I looked through those [the set of tasks] before we started this process, I was looking from what was the teacher was going to do....and what I would be performing, and that's what I wanted to change, I think. You know, is this my job or their job?...And I think the feeling about who's job it is has changed a little bit for me.

We had speculated, prior to the study, that engaging the teachers in learning about the Levels of Cognitive Demand would have some impact on their choices of categories for the final interview task-sorts. This, in fact, seems to have been the case. Twenty of the thirty-six categories from the last two task-sorts were based on the ideas the teachers had learned about the Levels of Cognitive Demand over the course of our work together. During the final interviews, the five teachers who used the Levels talked about their motivation for using those categories for this specific activity. Consider the following exchanges between the first author of this paper and individual teachers:

Fran: [Explain] how you sorted your tasks.
Craig: I went about and split it up. Started out in the high level and lower level tasks.
Fran: Okay...these are based on?
Craig: The sheet we had.
Fran: The sheet?
Craig: When I did it...I didn't have the sheet in front of me.
Fran: Okay, so this task sheet. [Indicating the Levels of Demand sheet]
Craig: Right....
Fran: Did you do it purposely because that's what we had worked on? Or was that just kind of the way you've started thinking about tasks?
Craig: I think that's just the way [I've] started thinking about it.

Fran: Talk to me about how you sorted these cards.
Brian: Okay. Well I figured I would try to do this [kind of] sort, but I wanted—
Fran: "This" kind of sort?
Brian: The one that we did early on in the study group.
Fran: Okay. Why did you figure you needed to try to do that?...I mean is that something that came to the top of your mind? Or did other things?...

Brian: When I thought about sorting, I thought well it, we use that to evaluate, assess what kinds of questions that were asked in tasks. So, I should try doing that.

Fran: What made you think about low, middle and high?
Ed: Well it made me think about it without actually getting out [the sheet], with some of the things we did here. [Indicating the Levels of Demand sheet] With higher level and lower level demands.

Fran: This second way that you sorted, you said referred back to this levels of demands that we've been talking about.
Carl: Uh huh.
Fran: Did you do it just because this is what we've been talking about? Or are you really thinking about tasks in this way now?
Carl: No, I really think about tasks in this way.
Fran: Okay. So you didn't do it like this because you thought 'Fran's coming and she wants me to do this.'
Carl: No, no.
Fran: Okay.
Carl: You wanted an honest interview. That's more what I thought.
Fran: Yes, I do.
Carl: So I didn't worry about that.

Megan: Well of course I mean it just seemed logical this time to...fit them into four different levels that we talked about of thinking.
Fran: Why did that seem logical?...
Megan: I think the first time we did this, that's [what] I was trying to do, but I didn't know how to say where I was putting them and so now I felt like I knew what my different areas should be, how to talk about them.
Fran: Okay.
Megan: How to think about them and then have an idea of what should [go] where and why?...
Fran: You said it seemed logical for this. Are you using these levels outside of doing this? I mean when you look at tasks, is that what comes to mind now?...
Megan: Yeah, it does. It definitely comes to mind more than it did before.

Their use of the Levels of Cognitive Demand to complete the final interview task-sorts together with the pieces of dialogue above provide some evidence that these five teachers had begun to use the Levels of Cognitive Demand to think about differences in the way a task is written, particularly for this activity (the task-sorts). But the dialogues above also hint at their use of the Levels of Cognitive Demand outside of this specific activity. Megan, Carl, and Craig all indicate that the Levels of Cognitive Demand are in their minds as they think about the tasks they use with their students. Are there other indicators of the use of Levels of Cognitive Demand outside of these card sorts? In the next section we discuss how the learning about the Levels of Cognitive Demand influenced, or in some cases did not influence, teachers' choices of tasks in their geometry classes.

Tasks That Teachers Used in Their Classrooms

Two week's worth of assignments. Results of analysis examining the mathematical tasks these five teachers used in their classrooms in two different weeks are contained in Table 7. Note that the first week was during October, 1999 and the second week was during March, 2000.

---Insert Table 7 about here---

In examining each teachers' two week's worth of assignments (one from the beginning of the study and one from the end of the study) we found no significant change in the *overall* characteristics of tasks that teachers assigned to their students over the course of this study. As a group, the number of tasks assigned increased from the first week to the second week (390 and 544 respectively), as did the number of high-level tasks (91 and 130 respectively). But upon closer examination, the percentage of high-level tasks to total number of tasks did not change. For the first week the percentage of high-level tasks used was 23.33%; for the second week 23.38%. Even given a margin of error for rating a few of the tasks incorrectly, the change would not be worth noting.

Examining the data on an individual basis provides a slightly different picture. Table 8 shows the percentage of high-level tasks teachers used per week. Carl, Ed, and Pamela each had little change in the percentage of high-level tasks from the 1st week to the 2nd week. Brian, Carl, and Megan showed considerable percentage gains. What do those percentage gains really mean? Are there really differences between the types of tasks (based on cognitive demand) that these teachers gave to their students at the beginning of the study versus those they gave at the end of the study? A closer examination of these three teachers' weeks' worth of tasks follows.

---Insert Table 8 about here---

Brian had the largest increase in percentage points (32% to 100%). During the examination of the actual tasks that he assigned, an explanation for this increase became very clear. Brian gave a test to his students in the middle of the first week, and two of the assignments that he gave his students prior to that test were meant for review, and contained only two high-level tasks (out of 22 total). For example, for review Brian asked his students to take a sentence like "if a triangle is equilateral, then it has all 60 degree angles" and determine whether it was true or false, underline the hypothesis once, and underline the conclusion twice. Then he asked the students to write the converse of the statement and identify the hypothesis and conclusion in the converse. Following the test, when his students were learning new material, Brian chose tasks that were all high-level. For example, one of the tasks in which Brian engaged his students involved used The Geometer's Sketchpad (Jackiw, 1997), a dynamic geometry software, to explore, conjecture, and test conjectures about parallel lines.

During the 2nd week, Brian gave no assessment and chose tasks that were all high-level, similar in nature to the high-level task described above, where students had to explore, conjecture, and then come to some conclusion. So, are his gains, as shown during this analysis, really true gains? We argue no. Brian was choosing high-level tasks at the beginning of this study for his students to engage in to *learn* mathematics and was doing the same at the end of the study.

In comparing Craig's tasks from the 1st and 2nd weeks, we did find a difference in the types of tasks he was choosing. During the 1st week, Craig chose mostly tasks from the school-adopted text (considered to be "traditional" in nature), and the one set of tasks that he chose from outside material contained mostly low level tasks. For example, one of the tasks that Craig chose, an "enrichment" task from the school-adopted text supplementary materials, was a geometry crossword puzzle. Students were required to fill in the crossword puzzle using clues such as "points on the same line are ____," and "the set of all points collinear to two points is a _____." In comparison, Craig began the 2nd week with an investigation from *Discovering Geometry* (Serra, 1997) in which his students had to generate definitions based on given information. This type of task, from the 2nd week, is a total departure from what Craig was engaging his students in during the 1st week. So, are Craig's gains, as shown during this analysis, really true gains? We argue yes. A difference exists, between the 1st week and the 2nd week, in the types of tasks Craig chose for his students to engage in while learning mathematics.

Megan's 1st week's worth of tasks all dealt with area of polygons and circles, and volume and surface area of solids (pyramid, prism, cone, and cylinder). Many of her tasks were similar, but with different figures. For example, she would provide a picture of a rectangle with the measurement of two dimensions shown, and then ask her students to find the area of the rectangle. Megan had given her students three worksheets containing a number of problems that were this type of task. When Megan was asked where these worksheets originated, she described them as "just ones I made up." The high-level tasks that Megan gave her students in the 1st week came from her classroom text, *Discovering Geometry* (Serra, 1998), and included a project in which students were required to look at a floor plan for a house and research the costs of products with which to build the house. During the 2nd week, Megan's students were learning about triangle congruency and were engaged in tasks that all originated in the *Discovering Geometry* text. As indicated in Figure 7, 73% of those tasks can be categorized as requiring a high-level of cognitive demand.

Megan herself seemed to be aware of the low-level nature of the tasks that she had chosen during the 1st week. Because of scheduling conflicts, Megan was not able to do her initial interview until after the first study group meeting (although she had done the task-sorts and chosen her 1st week's worth of activities before the day of the first meeting). Consequently, when she was asked about her 1st week's worth of assignments during the interview, she was able to reflect upon them using the knowledge about the Levels of Cognitive Demand criteria she had gained during the first study group meeting:

Fran: If you took those problems and you put them on a scale from 1 being "these problems don't help my kids reason at all" to 10 being "wow, these problems really help my kids with their reasoning ability and to reason about geometry," where would your problems fall on that continuum?

Megan: Probably pretty low at this point. I think by the time we get to the final project I'm having them do...they'll be up there higher because I'll have them really go through and do a lot of explanations. I look back at some of those problems I gave you and I thought oh, first [there are] problems to make sure they understand the formulas, you know there wasn't much with that. Some of the other problems about...having them find the little areas they can paint—I don't really have them go through and do explanations and stuff, and I could have gone back through and had them do that and that would have made them maybe reason a little bit more or be able to explain the reasoning. Which I think would have been good so they would be more prepared to do a bigger project. So, looking back, they weren't that high.

During her final interview, Megan was asked the same question about the tasks that she had recently chosen to use with her students:

> Fran: On a scale from 1, "these tasks are not helping my students with their reasoning at all", to 10, "these tasks are helping my students a lot with their reasoning"...where do you think you would rank the kinds of tasks that you pick to use in class?...
>
> Megan: At this point I think I've improved a lot from what I was.

So, are Megan's gains, as shown during this analysis, really true gains? We argue yes. A difference exists, between the 1st week and the 2nd week, in the types of tasks Megan chose for her students to engage in while learning mathematics.

Thus far, our reported results indicate that Brian consistently used high-level tasks over the course of the study, and that Megan and Craig increased their use of high-level tasks over the course of the study. Does that mean that the other four teachers (Annie, Carl, Ed, and Pamela) were not influenced by their participation in this study group to integrate more high-level tasks into their practice?

Evidence (too lengthy to include in this paper) does exist elsewhere (Arbaugh, 2000) that three out of these four teachers (Annie, Carl, and Pamela) did in fact integrate high-level tasks in their classrooms over the course of this study. Annie was constantly looking for "better" tasks to use with her students. During her initial interview, Annie talked about her frustration with the level of tasks contained in the adopted text for her course:

> Annie: Well one thing that I could say is that for the beginning part of the unit where we're working on inductive and deductive reasoning and getting heading into proofs and looking at proofs, I didn't use a big chunk of the book because the book problems were so low order and so confusing and just so awful.
> Fran: What do you mean so low order?
> Annie: A lot of true false things a lot of no asking for explanation there's no way to tell does this person actually understand what they're doing or not or are they just guessing.

Annie brought many higher-level tasks to study group to share with her colleagues and was always ready to try out activities in her classroom from outside resources. Data collected regarding her use of supplementary tasks over the course of the study shows that she was integrating day-long high-level supplementary activities into her geometry classrooms 2 to 3 times a month.

Carl, who used *Discovering Geometry* as a text in his geometry classes, was also implementing high-level tasks at a consistent rate throughout the study. He worked solely out of the text, which was written so as to support student conjectures, testing of those conjectures, and then making generalizations (characteristics of high-level tasks).

Evidence exists that Pamela was also integrating high-level tasks into her geometry classes, although more towards the end of the study than towards the beginning. She provides an interesting case of a teacher who began the study having to be convinced of the worth of a supplementary task or activity before being willing to implement it with her students, and ending the study much more willing to design and implement tasks that did not come from her text. For example, early in the process, four of the teachers volunteered to "pilot" an activity from *Connected Geometry* (Education Development Center, 2000); Pamela was not one of those four teachers. At the study group meeting following the "piloting" of the activity (The Envelope Game), the teachers discussed what happened during implementation. Pamela listened to her colleagues and then the following exchange occurred:

> Pamela: I was very skeptical about this activity, just because I didn't know how much they [the students] were really going to benefit from it. So, when I looked at it I thought well, is it worth the time that it takes for me to plan this and have this great activity to give for them and spend a couple of days on it and hope that they get a much better understanding? Or is it better to tell them and have discussions, I

guess, as to why these work and don't. And so I'm looking very forward to doing this next year.

Fran: So this has changed your mind?

Pamela: Yeah....I was really skeptical so I didn't do it, and now that I see that it worked and I know what didn't work I'd say I'm more likely to use this next year. I was just really skeptical as to whether this really would benefit all of them. Or if, a lot of times when I have them work in groups to do things like this they think they are not learning any more. It's just something that they are doing in groups, so I didn't want them to look at it that way. So that's something I have to improve on I think in my classes anyway. Is trying to get them to realize that group work is still learning. It's not.

After this meeting, Pamela began supplementing the adopted text with high-level tasks on occasion. She became much more willing to try a task or activity that her colleagues brought to study group meetings, and even began designing some supplementary tasks that other teachers also used in their classes.

Ed was the only teacher for which we could not find the evidence to argue for his continual use of high-level tasks in his geometry classes.

Evidence also exists (Arbaugh, 2000) that some of the teachers who were integrating more high-level tasks in their classrooms were doing so as a result of learning about the Levels of Cognitive Demand. Megan, for example, talked above about how learning about the Levels caused her to reflect upon the tasks she was using with her students. Annie, who first learned about the Levels of Cognitive Demand in her mathematics methods class, attributed her desire to implement high-level tasks directly to having knowledge of the Levels. Consider this piece of transcript from her final interview:

Annie: Well when I look through a book problem I do think about, is it a skill or is this problem solving? That sort of thing. And I do tend to look at problems that way....

Fran: Do you think you'd be thinking about them as skill or problem solving oriented if we hadn't talked about the Levels of Cognitive Demand this year?

Annie: If I hadn't done it in methods class I would not know – I'd just pick some of these and some of these [pointing to the different sections of the text problem set].
Fran: Okay. So you go back to methods class?
Annie: Yeah.

Not all of the teachers directly attributed their use of high-level tasks in their geometry classes to learning about the Levels of Cognitive Demand. But those teachers who did begin to supplement their adopted "traditional" text with more high-level tasks did attribute it to participation in the study group, where the Levels of Cognitive Demand were a major focus. For example, in her final interview, Pamela talked about the influence the study group had on her choice of tasks:

Fran: [You've said] "I tend to bring in more outside material than I might have if I'd not been in this study group." Talk to me about that.
Pamela: I know I do. Because I know before this -- and part of it is because of seeing how the other, how fast the other teachers go through the book. I was way ahead, because I was just doing what's in the book. Here's what we've got to learn. Here's how we're going to learn it, and we did it and that's all there is. I don't know if I would of thought um, without this study group that 'hey, I think I need to bring in some other stuff here.' I imagine in some of these sections I would of, because some it just isn't supportive of what's in there, I think. But I think this helped me to say, you know, bring in some other activities that will help them see this a little bit different, you don't always have to use the book.
Fran: And you attribute that to being in study group?
Pamela: Uh huh [yes].

While the teachers in this study demonstrated that they had the ability to choose high-level tasks to use in their geometry classrooms, and some began to incorporate more high-level tasks into their teaching, we do not know how the tasks were set up or implemented in their classrooms. Set up and implementation (two phases of the Mathematical Tasks Framework) are important next steps to investigate with teachers who learn about the Levels of Cognitive Demand

Discussion and Conclusion

Over the course of this study, the teachers in this study were engaged in collegial discussions during which they thought deeply about the mathematical tasks they implemented in their geometry classes. The Levels of Cognitive Demand criteria from the Mathematical Tasks Framework provided one of the framing concepts for the professional development of these teachers. Results reported in this paper indicate that our work with the Levels of Cognitive Demand influenced how these teachers thought about instructional planning and implementation issues in the following manners:

- The teachers began to think more deeply about the relationship between tasks and levels of student thinking (or reasoning);
- Some of the teachers began to select and incorporate more high-level tasks into their teaching practices.

The Mathematical Tasks Framework served a vital role in influencing these teachers' thinking about mathematical tasks. This framework was developed through work done with middle grades students, and theoretically based, in part, on work done with elementary grades students. Stein et al. (2000) assert that the Mathematical Tasks Framework is a useful framing concept for providing quality professional development for inservice mathematics teachers, specifically at the middle school level. The research reported in this study furthers the field by providing evidence indicating that the Levels of Cognitive Demand criteria from the Mathematical Tasks Framework is also an appropriate framing concept for professional development at the high school level.

Evidence reported elsewhere (Arbaugh, 2000) further indicates that the Mathematical Tasks Framework in its entirety (focusing not only on the Levels of Cognitive Demand of tasks as written in the curriculum but also on the set up and implementation of those high level tasks) provides teachers with support for thinking about instructional decisions and/or implementation issues. Designing professional development for teachers is a challenging task and the diverse needs of teachers at different stages of their careers are important for inservice teacher educators to consider:

> The first few years of teaching present a very different period in the professional development process. Initial teaching assignments and support structures play a significant role in shaping beginning teachers' views of the profession and their commitments to it....New issues are confronted, and knowledge and skills are built daily, more often within the context of the teaching environment than through formal continuing education. As teachers of mathematics become more experienced, collegial interactions increase and teachers assume a different role....experienced teachers may become mentors to beginning or developing teachers at this time in their careers....Their ability to engage in ongoing analysis of their own teaching and learning is often central to their seeking experiences that address knowing mathematics, knowing students, and knowing teaching. (NCTM, 1991, p. 124)

The Mathematical Tasks Framework provides guidance related to the areas of knowledge and skill building. It also provides a framework that encourages self-reflection and self-analysis. We have found the diversity of support provided by this framework to be a strength of using the Mathematical Tasks Framework with these teachers. Further longitudinal research should be undertaken in which the Mathematical Tasks Framework is used as a framing concept for the professional development of high school mathematics teachers. Future investigations should also include questions regarding the influence that the Mathematical Tasks Framework has on teachers' questioning and assessment practices.

This study also supports the call for investigating new frameworks that allow us to understand the work of teacher education researchers – scholarship that Zeichner (1999) describes as "likely to enliven and stimulate us into seeing the familiar in new ways and into seeing patterns, relationships, and connections for the first time" (p. 12). This study suggests that the Mathematical Tasks Framework can be a useful research tool for understanding in-service mathematics teacher education and how it relates to teacher knowledge and practice. For example, it guided this particular research by providing a framework that influenced research questions, decisions about data collection and analysis, and ultimately how we interpreted and reported the results of this study (Eisenhart, 1991). The need for further investigation into the use of this framework as a research tool exists, as important questions about its use arose from this study. For example, "Is the Mathematical Tasks Framework an appropriate research tool if the framework itself is not used as a framing concept for the professional development?"

McGraw, Lynch, Koc, Kapusuz, and Brown (2002) begin to address this question by using the Mathematical Tasks Framework as a tool for analyzing and describing teacher talk during small group discussions. Their use of the Mathematical Tasks Framework suggests that the tentative answer to the above question is "yes;" further investigation into this question is warranted. Does this framework allow us to understand the work of teacher educators, as Zeichner (1999) describes? What doesn't the framework allow us to see?

References

Arbaugh, F. (in review). Study groups as a form of professional development for secondary mathematics teachers.

Arbaugh, F. (2000). *Time on tasks: Influences of a study group on high school mathematics teachers' knowledge, thinking, and teaching.* Unpublished doctoral dissertation, Indiana University, Bloomington.

Arbaugh, F., & Brown, C. A. (in review). What makes a mathematical task worthwhile? Designing a learning tool for high school mathematics teachers

Coffey, A., & Atkinson, P. (1996). *Making sense of qualitative data: Complementary research strategies.* Thousand Oaks, CA: Sage Publications.

Doyle, W. (1983). Academic work. *Review of Educational Research, 53*(2), 159-199.

Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational Psychologist, 23* (2), 167-180.

Education Development Center, Inc. (EDC). (2000). Connected geometry. Chicago: Everyday Learning Corporation.

Eisenhart. M., (1991). Conceptual frameworks for research circa 1991: Ideas from a cultural anthropologist; implications for mathematics education researchers. *Proceedings of the Thirteenth Annual Meeting of the Psychology of Mathematics Education - North America: Plenary Papers & Reactions, Symposia & Papers*, (p. 202). Christiansburg, VA: Christiansburg Printing Company, Inc.

Ernest, P. (1997). The epistemological basis of qualitative research in mathematics education: A postmodern perspective. In A. R. Teppo (Ed.) *Qualitative research methods in mathematics education* (Journal for Research in Mathematics Education Monograph No. 9, pp. 22-39). Reston, VA: National Council of Teachers of Mathematics.

Fennema, E., & Nelson, B. S. (Eds.). (1997). Mathematics teachers in transition. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28* (5), 524-549.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal, 30*, 393-425.

Jackiw, N. (1997). The geometer's sketchpad [Computer software]. Berkeley, CA: Key Curriculum Press.

Marx, R. W., & Walsh, J. (1988). Learning from academic tasks. *The Elementary School Journal, 88* (3), 207-219.

McGraw, R., Lynch, K., Koc, Y., Kapusuz, A., & Brown, C. A. (2002). Talking about mathematics teaching: An examination of the use of a multi-media case to stimulate the conversation. Paper presented at the Annual Meeting of the American Educational Research Association, April 2002, New Orleans.

National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics*. Reston, VA: Authors.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Authors.

Serra, M. (1997a). *Discovering geometry: An inductive approach*, 2nd Edition. Emeryville, CA: Key Curriculum Press.

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*(5), 344-350.

Stein, M. S., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455-488.

Stein, M. S., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2,* 50-80.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. NY: Teachers College Press.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268-275.

Van de Walle, J. A. (1994). *Elementary school mathematics: Teaching developmentally*, 2nd edition. White Plains, NY: Longman.

Zeichner, K. (1999). The new scholarship of teacher education. *Educational Researcher, 28* (9), 4-15.

Table 1: Levels of Cognitive Demand

Levels of Demands*

Lower-level demands (memorization):

•Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory.
•Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
•Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
•Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced.

Lower-level demands (procedures without connections to meaning):

•Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
•Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
•Have no connection to the concepts or meaning that underlie the procedure being used.
•Are focused on producing correct answers instead of on developing mathematical understanding.
•Require no explanations or explanations that focus solely on describing the procedure that was used.

Higher-level demands (procedures with connections to meaning):

•Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
•Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
•Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
•Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (doing mathematics):

•Require complex and nonalgorithmic thinking -- a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
•Require students to explore and understand the nature of mathematical concepts, processes, or relationships.
•Demand self-monitoring or self-regulation of one's own cognitive processes.
•Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
•Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
•Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

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Table 1. Directions for initial and final interview task-sorting activity

Directions for Task-Sorts
This packet contains 20 middle-school level mathematics tasks. • Please read them through, and then sort them into categories of your own making. Write down the names of your categories and then list the tasks that fall into each category. • Could the tasks be sorted in a different way? Do a second sort with different categories than you used before. Write down the names of your categories and then list the tasks that fall into each category. Please be prepared to talk to me about your categories and task placement decisions when we meet for your interview.

Table 3. Emergent Categories from Teacher Task Sorts

Emergent Categories	Initial Interview	Final Interview
Categories based on mathematical content or topics	X	
Categories based on what students have to do to complete the task (processes)	X	X
Categories based on what students have to do to answer the task (end-result)	X	X
Categories based on surface characteristics of tasks as presented (based in text and visuals)	X	X
Other	X	
Categories based on Levels of Cognitive Demand		X

Table 4. Dates of Initial and Final Task Sorts

Interview	**Date of Interview**	**Task Sort Completed**
Initial Interviews	October 1999	Initial Sort 1 Initial Sort 2
Final Interviews	March 2000	Final Sort 1 Final Sort 2

Table 5. Emergent Groups from Initial Interview Task Sorts

Initial Interview Task Sorts 1 and 2	
Names of Emergent Groups	# of Teacher-Created Categories per Group
Categories based on mathematical content or topics	16
Categories based on what students have to do to complete the task (processes)	5
Categories based on what students have to do to answer the task (end-result)	3
Categories based on surface characteristics of tasks as presented (based in text and visuals)	27
Other	3

Table 6. Emergent Groups from Final Interview Task Sorts

Final Interview Task Sorts 1 and 2	
Names of Emergent Groups	# of Teacher-Created Categories per Group
Categories based on the Levels of Cognitive Demand	20
Categories based on what students have to do to complete the task (processes)	4
Categories based on what students have to do to answer the question (end-result)	2
Categories based on surface characteristics of tasks as presented (based in text and visuals)	7

Table 7. Number of tasks used by each teacher (by week) and cognitive levels of demands required by those tasks

	1st Week			2nd Week		
	Total # of Tasks	High-Level	Low-Level	Total # of Tasks	High-Level	Low-Level
Annie	7	1	6	*	*	*
Brian	31	10	21	26	26	0
Carl	103	55	48	41	20	21
Craig	62	4	58	72	19	53
Ed	98	2	96	137	0	137
Megan	52	17	35	84	61	23
Pamela	37	2	35	184	4	180

*indicates incomplete data

Table 8. Percentage of High-Level Tasks Used

Name	1st Week	2nd Week
Brian	32	100
Carl	53	49
Craig	6	26
Ed	2	0
Megan	32	73
Pamela	6	2

Figure 1. Mathematical Tasks Framework*

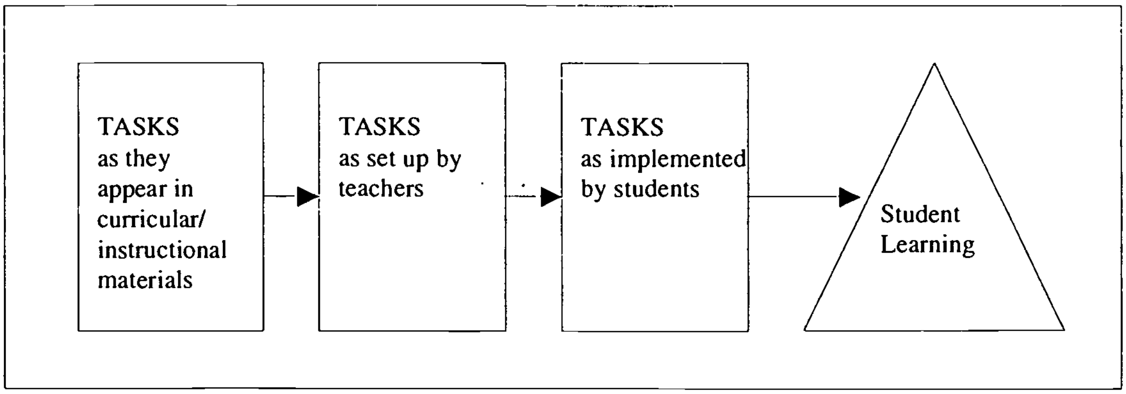

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Figure 2. Number of Categories in Initial Sorts 1 and 2 by Teacher

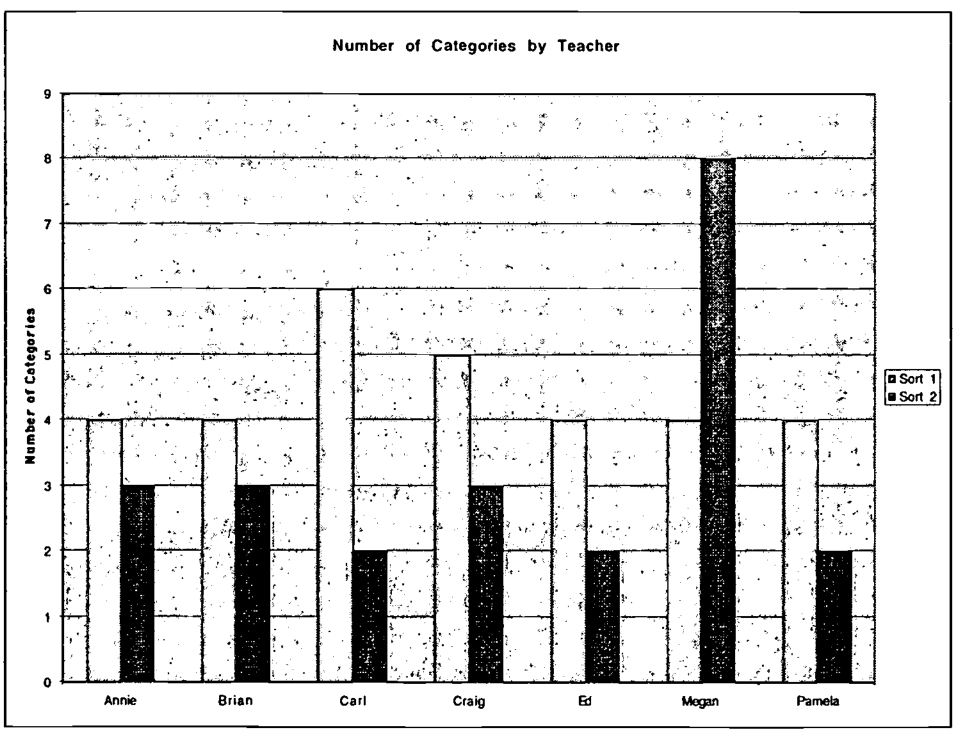
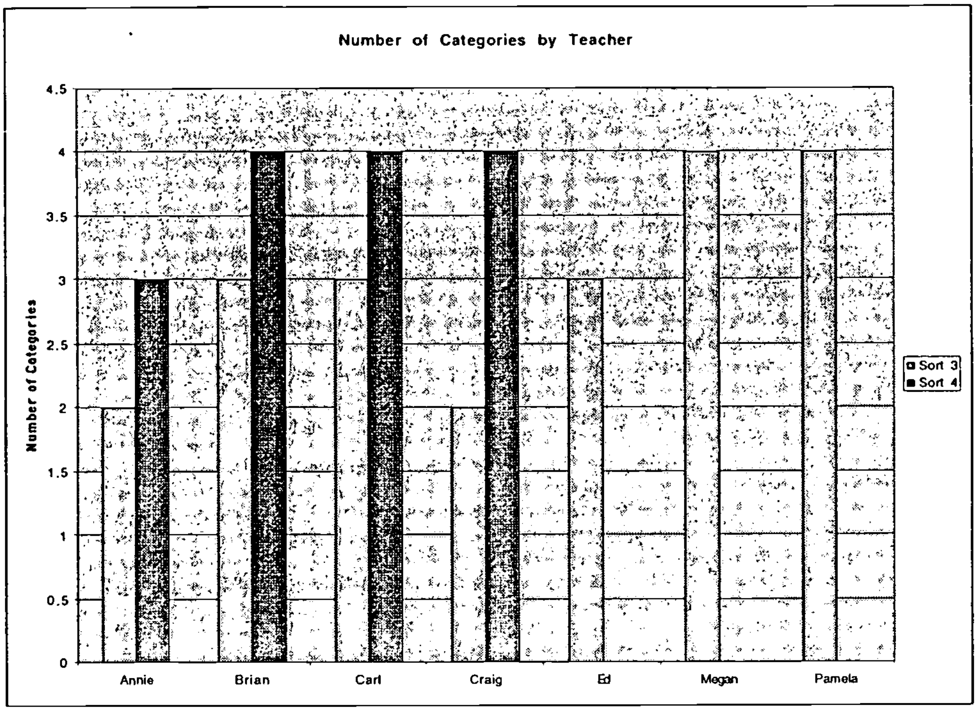


Figure 3. Number of Categories in Final Sorts 1 and 2 by Teacher



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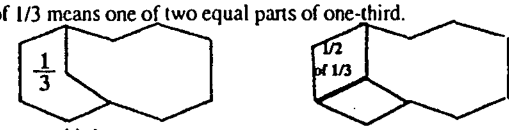
TASK A

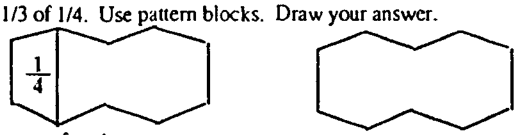
Manipulatives/Tools: Pattern Blocks
Level: Middle School
Task:

1/2 of 1/3 means one of two equal parts of one-third.



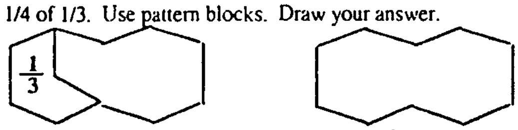
one-third   1/2 of 1/3 or 1/2 x 1/3 = 1/6

Find 1/3 of 1/4. Use pattern blocks. Draw your answer.



one-fourth   1/3 of 1/4 or 1/3 x 1/4 = $\square$

Find 1/4 of 1/3. Use pattern blocks. Draw your answer.

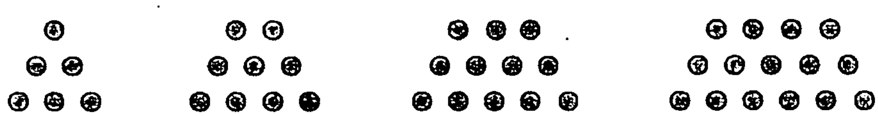


one-third   1/4 of 1/3 or 1/4 x 1/3 = $\square$

TASK B

Manipulatives/Tools Available: Counters
Level: Middle School
Task:

For homework Mark's teacher asked him to look at the pattern below and draw the figure that should come next.



Mark does not know how to find the next figure.

A. Draw the next figure for Mark.

B. Write a description for Mark telling him how you knew which figure comes next.

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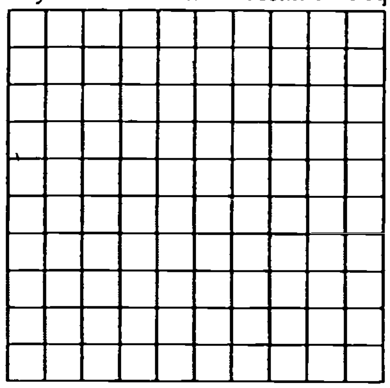
TASK C

Manipulatives/Tools: 10 x 10 square grid
Level: Middle School
Task:

Shade 1/3 of the decimal square and express as a decimal. Use the decimal square to explain to your partner why the fraction and the decimal are equivalent.



TASK D

Manipulatives/Tools: None
Level: Middle School
Task:

Evaluate each expression when $x = 3$

$3x$
$5 + x$
$x + 21$
$18 - x$
$x \div 6$

TASK E

Manipulatives/Tools: Calculator
Level: Middle School
Task:

Divide using paper and pencil. Check your answer with a calculator and round the decimal to the nearest thousandth.

$\frac{525}{1.3}$

$\frac{52.75}{7.25}$

$\frac{30.459}{.12}$

TASK F

Manipulatives/Tools: None
Level: Middle School
Task:

Match the property name with the appropriate equation.

1. Commutative property of addition
2. Commutative property of multiplication
3. Associative property of addition
4. Associative property of multiplication
5. Identity property of addition
6. Identity property of multiplication
7. Inverse property of addition
8. Inverse property of multiplication
9. Distributive property
10. Property of zero for multiplication

a. $3(s+t) = 3s = 3t$
b. $5 \cdot 1/5 = 1$
c. $-4 + x = x + (-4)$
d. $2/3 + -2/3 = 0$
e. $8 \cdot (2x) = (8 \cdot 2)x$
f. $1 \cdot (xy) = xy$
g. $9 \cdot 0 = 0$ and $0 \cdot 9 = 0$
h. $x + (b + 4) = (x + b) + 4$
i. $3y + o = 3y$
j. $7 \cdot 5 = 5 \cdot 7$

TASK G

Manipulatives/Tools Available: Base Ten Blocks, grid paper
Level: Middle School
Task:

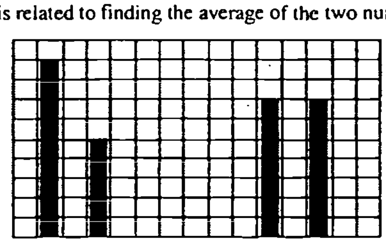
.08 .8 .080 .008000

Make three observations about the relative size of the above 4 numbers. Be sure to explain your observations as clearly as possible. Feel free to illustrate your observations if you feel it would help others understand them.

Adapted from QUASAR Project--QUASAR Cognitive Assessment Instrument--Release Task

TASK H

Manipulatives/Tools: Grid Paper
Level: Middle School
Task:
The pairs of numbers in a - d below represent the heights of stacks of cubes to be leveled off. On grid paper, sketch the front views of columns of cubes with these heights before and after they are leveled off. Write a statement under the sketches that explains how your method of leveling off is related to finding the average of the two numbers.



a) 14 and 8 b) 16 and 7 c) 7 and 12 d) 13 and 15

By taking 2 blocks off the first stack and giving them to the second stack, I've made the two stacks the same. So the total # of cubes is now distributed into 2 columns of equal height. And that is what average means.

Taken from <u>Visual Mathematics</u> (Bennett & Foreman, 1989)

TASK I

Manipulatives/Tools: None
Level: Middle School
Task:

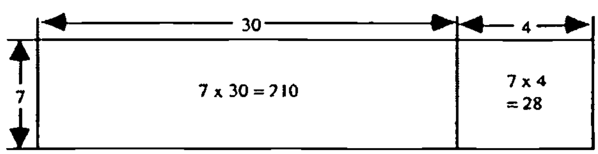
Write and solve a proportion for each.

17 is what percent of 68?
What is 15% of 60?
8 is 10% of what number?
24 is 25% of what number?
28 is what percent of 140?
What is 60% of 45?
36 is what percent of 90.
What is 80% of 120?
21 is 30% of what number?

TASK J

Manipulatives/Tools: None
Level: Middle School
Task:

One method of mentally computing 7 x 34 is illustrated in the diagram below:



Mentally compute these products. Then sketch a diagram that describes your methods for each.
a) 27 x 3

b) 325 x 4

Taken from <u>*Visual Mathematics*</u> *(Bennett & Foreman, 1989).*

TASK K

Manipulatives/Tools: None
Level: Middle School
Task:

The Georgetown University basketball team had a total of 252 free throws in 14 games. Find the average number of free throws per game.

TASK L

Manipulatives/Tools: Base-10 Blocks
Level: Middle School
Task:

Using Base-10 blocks, how could you show that 0.292 is less than 0.3?

TASK M

Manipulatives/Tools Available: None
Level: Middle School
Task:

Write three different mathematical problems that can be solved using the information below.

Jerome, Elliot, and Arturo took turns driving home from a trip. Arturo drove 80 miles more than Elliot. Elliot drove twice as many miles as Jerome. Jerome drove 50 miles.

Question #1

Question #2

Question #3

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TASK N

Manipulatives/Tools: None
Level: Middle School
Task:

The cost of a sweater at J. C. Penney's was $45.00. At the "Day and Night Sale" it was marked 30% off of the original price. What was the price of the sweater during the sale? Explain the process you used to find the sale price.

TASK O

Manipulatives/Tools: None
Level: Middle School
Task:

Give the fraction and percent for each decimal.

.20 = ____=____

.25 = ____=____

$.3\bar{3}$ = ____=____

.50 = ____=____

$.6\bar{6}$ = ____=____

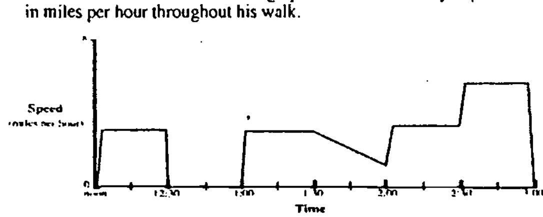
.75 = ____=____

TASK P

Manipulatives/Tools Available: None
Level: Middle School
Task:

Use the following information and the graph to write a story about Tony's walk:

At noon. Tony started walking to his grandmother's house. He arrived at her house at 3:00. The graph below shows Tony's speed in miles per hour throughout his walk.



Write a story about Tony's walk. In your story, describe what Tony might have been doing at the different times.

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TASK Q

Manipulatives/Tools Available: Calculator with scientific functions
Level Middle School
Task:

Penny's mother told her that several of her great-great-great-grandparents fought in the Civil War. Penny thought this was interesting and she wondered how many great-great-great grandparents that she actually had. When she found that number, she wondered how many generations back she'd have to go until she could count over 100 ancestral grandparents or 1000, or 10,000, or even 100,000. When she found out she was amazed and she was also pretty glad she had a calculator. How do you think Penny might have figured out all of this information? Explain and justify your method as clearly and completely as possible.

Adapted from *<u>Mathematics with Calculators: Resources for Teachers, Grade 6</u>*, *Section 3, Activity 2. Addision-Wesley Publishing Company, 1989.*

TASK R

Manipulatives/Tools: None
Level: Middle School
Task:

Part A: After the first two games of the season, the best player on the girl's basketball team had made 12 out of 20 free throws. The best player on the boys' basketball team had made 14 out of 25 free throws. Which player had made the greater percent of free throws?

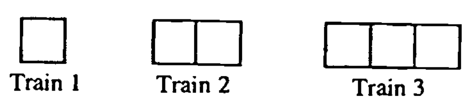
Part B: The "better" player had to sit out the third game due to an injury. How many baskets (out of an additional 10 free throw "tries") would the other player need to make in order take the lead in terms of greatest percentage of free throws?

Adapted from Investigating Mathematics, Gencoe Macmillan/McGraw-Hill, New York, New York, 1994.

TASK S

Manipulatives/Tools: Square Pattern Tiles
Level: Middle School
Task:

Using the side of a square pattern tile as a measure, find the perimeter (i.e., distance around) of each train in the pattern block figure shown below.



TASK T

Manipulatives/Tools Available: Calculator
Level Middle School
Task:

Your school's science club has decided to do a special project on nature photography. They decided to take a little over 300 outdoor photos in a variety of natural settings and in all different types of weather. Eventually they want to organize some of the best photos into a display and enter the State nature photography contest. The club was thinking of buying a 35mm camera, but someone in the club suggested that it might be better to buy disposable cameras instead. The regular camera with autofocus and automatic light meter would cost about $40.00 and film would cost $3.98 for 24 exposures and $5.95 for 36 exposures. The disposable cameras could be purchased in packs of three for $20.00 with two of the three taking 24 pictures and the third one taking 27 pictures. Single disposables could be purchased for $8.95. The club officers have to decide which would be the best option and they have to justify their decisions to the club advisor. Do you think they should purchase the regular camera or the disposable cameras? Write a justification that clearly explains your reasoning.

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